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T VI: Nonlinear Dynamics and Complex Systems

(Prof. E. Frey)

Problem Set 10:
Superpositions of stripe states

Problem 1 *Stripe – lattice interaction*

Consider modes equal to the critical wavenumber only. Then you are allowed to drop the spatial derivatives in the corresponding amplitude equations. Use the models given below as well as the general amplitude formalism to compute the function $G(\theta)$ for

a) the standard Swift-Hohenberg equation

$$\partial_t u = ru - (1+\nabla^2)^2 u - u^3$$

b) and the modified Swift-Hohenberg equation

$$\partial_t u = ru - (1+\nabla^2)^2 u + \nabla \cdot ((\nabla u)^2 \nabla u).$$

c) Conclude from the general observations in the lecture for the amplitude equation (with the symmetries corresponding to the models above) which state, stripe or lattice, is stable!

Problem 2 *Hexagonal Patterns*

A hexagonal pattern requires a superposition of three modes:

$$u(x,y) = e^{i(\vec{q}_1\cdot\vec{x}+\Phi_1)} + e^{i(\vec{q}_2\cdot\vec{x}+\Phi_2)} + e^{i(\vec{q}_3\cdot\vec{x}+\Phi_3)} + c.c.$$

with $\vec{x} = (x,y)$ and the wave vectors $\vec{q}_1$, $\vec{q}_2$ and $\vec{q}_3$ forming an equilateral triangle

$$\vec{q}_1 + \vec{q}_2 + \vec{q}_3 = 0 \quad \text{and} \quad |\vec{q}_i| = 1 \quad \forall i,$$

and where the Φ_i are the possible phases of the modes. Since q sets the scale of the pattern, let's choose $q = 1$, as mentioned, and orient our axes so that $\vec{q}_1 = (1,0)$.

a) What is $\vec{q}_2$ and $\vec{q}_3$?

b) Show that by redefining the origin of the coordinates we may substitute Φ_1 and Φ_2 such that the pattern is determined by a single phase variable Φ only.

c) Plot $u(x,y)$ with the help of mathematica or similar plotting programs. Do you get a hexagonal pattern for an arbitrary choice of Φ?

Problem 3 *Stability of a hexagonal state*

Again, consider critical modes only.

a) Write down the three amplitude equations describing the emergence of a hexagonal state. What is the value of θ in $G(\theta)$ for hexagonal patterns? Determine the amplitude of the hexagonal pattern A_H (for this use $A_1 = A_2 = A_3 = A_H$)!

b) Derive the condition for stability of the hexagonal state $A_1 = A_2 = A_3 = A_H$ by using the corresponding amplitude equations!

c) Analogously, show that stripes ($A_1 = A_s, A_2 = A_3 = 0$) are unstable for $G < 1$ and stablet for $G > 1$!