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3rd Exercise Sheet Many-Body Physics

Will be discussed in the week of May 13-17.

Exercise 1: Analytic Continuation

Consider the integral $I(z) = \int_{-\infty}^{\infty} \frac{dp}{p^2+z}$ for complex z.

- Compute the integral for real positive $z = \rho > 0$.
- Use the previous result as a starting point for analytic continuation to extend $z = \rho e^{i\phi}$ to the entire complex plane, with ϕ the phase.
- Explain why your answer is not unique, and explain the difference between $\int_{-\infty}^{\infty} \frac{dp}{p^2-\rho-i\eta}$ and $\int_{-\infty}^{\infty} \frac{dp}{p^2-\rho+i\eta}$.

Exercise 2: Non-interacting Green functions for fermions at zero temperature

- Derive the non-interacting fermionic Green function in momentum space directly by Fourier transform of the non-interacting Green function in real space-time.
- Show that the Lehmann representation introduced for the fermionic interacting many-particle system reduces to the Green function in momentum space for non-interacting fermions in the absence of interactions.
- Show that the fermionic non-interacting Green function reproduces the correct density and energy.

Exercise 3: Lehmann representation (see FW 3.8)

Derive the Lehmann representation for $D(\boldsymbol{k}, \omega)$, which is the Fourier transform of

$$iD(x, y) = \frac{\langle\Psi_0|T[\tilde{n}_H(x)\tilde{n}_H(y)]|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}. \tag{1}$$

with the density fluctuation operator defined by

$$\tilde{n}(\boldsymbol{x}) = \psi^\dagger_\alpha(\boldsymbol{x})\psi_\alpha(\boldsymbol{x}) - \frac{\langle\Psi_0|\psi^\dagger_\alpha(\boldsymbol{x})\psi_\alpha(\boldsymbol{x})|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle} \tag{2}$$

Show that $D(\boldsymbol{k}, \omega)$ has poles in the second and fourth quadrant of the complex ω plane.