LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

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4th Exercise Sheet Many-Body Physics

Will be discussed in the week of May 20-24.

Exercise 1: Polarization

Consider the same operator as in exercise 3.3,

$$iD(x,y) = \frac{\langle \Psi_0 | T[\tilde{n}_H(x)\tilde{n}_H(y)] | \Psi_0 \rangle}{\langle \Psi_0 | \Psi_0 \rangle}. \tag{1}$$

Apply Wick's theorem, and show that it reduces to the polarization insertion (up to a factor of $\hbar$).

Exercise 2: Fermi system (FW 4.1)

A uniform spin-1/2 Fermi system has a spin-independent interaction potential $V(\boldsymbol{x}) = V_0 \frac{e^{-x/a}}{x}$.

a. Evaluate the self-energy in the Hartree Fock approximation. Find the excitation spectrum ϵ_k and the Fermi energy $\epsilon_{\mathrm{F}} = \mu$.

b. Show that the exchange contribution to ϵ_{F} is negligible for a long-range interaction ($k_{\mathrm{F}}a \gg 1$) but that the direct and exchange terms are comparable for a short-range interaction ($k_{\mathrm{F}}a \ll 1$).

c. In this approximation prove that the effective mass m^* is determined solely by the exchange contribution. Compute m^*, and discuss the limiting cases $k_{\mathrm{F}}a \gg 1$ and $k_{\mathrm{F}}a \ll 1$.

Exercise 3: Electron gas (FW 4.9)

Show that the selfenergy $\Sigma(\boldsymbol{q})$ of a degenerate electron gas is given to first order in the interaction by

$$\hbar\Sigma^{(1)}(\boldsymbol{q}) = -\frac{e^2}{2\pi}\left(\frac{k_F^2 - q^2}{q}\ln\frac{|k_F + q|}{|k_F - q|} + 2k_F\right). \tag{2}$$

Sketch the resulting single-particle spectrum. Discuss the effective mass $m^*(q)$ defined by $m^*(q) = (\hbar^2 q)(\partial\epsilon_q/\partial q)^{-1}$.