LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
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5th Exercise Sheet Many-Body Physics

Will be discussed in the week of May 27-31.

Exercise 1: Frequency summations

Show that

$$\sum_n \frac{e^{i\omega_n \eta}}{i\omega_n - x} = \frac{\pm\beta\hbar}{2\pi i} \int_C \frac{dz}{e^{\beta\hbar z} \mp 1} \frac{e^{\eta z}}{z - x}, \tag{1}$$

where the upper sign holds for bosons (even frequencies) and the lower signs for fermions (odd frequencies) and the contour C encircles the imaginary axis.

Show that

$$\lim_{\eta\to 0} \sum_n \frac{e^{i\omega_n \eta}}{i\omega_n - x} = \mp \frac{\beta\hbar}{e^{\beta\hbar x} \mp 1}, \tag{2}$$

Exercise 2: Frequency summations ($\hbar = 1$)

Show that

$$\frac{1}{\beta} \sum_{ip_n} G^0(\boldsymbol{p}, ip_n) G^0(\boldsymbol{k}, ip_n + i\omega_n) = \frac{n_F(\xi_p) - n_F(\xi_k)}{i\omega_n + \xi_p - \xi_k} \tag{3}$$

and

$$-\frac{1}{\beta} \sum_{ip_n} G^0(\boldsymbol{p}, ip_n) G^0(\boldsymbol{k}, i\omega_n - ip_n) = \frac{1 - n_F(\xi_p) - n_F(\xi_k)}{i\omega_n - \xi_p - \xi_k} \tag{4}$$

where G^0 is the non-interacting fermionic Green function, $n_F(\xi) = \frac{1}{e^{\beta\xi}+1}$ and ξ_k the single particle dispersion.