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6th Exercise Sheet Many-Body Physics

Will be discussed in the week of June 3-7.

Exercise 1: Feynman rules in imaginary time for the Green function

The goal of the exercise is to formally derive the Feynman rules for perturbation theory in the imaginary time formalism for a spin-independent instantaneous two-particle interaction $V(\boldsymbol{x}_1 - \boldsymbol{x}_2)$ and a quadratic non-interacting part H_0. We will assume that the system is translationally invariant and take the limit $V \to \infty$.

- Introduce for any operator $\hat{O}_S$ in the Schrödinger picture the interaction picture $\hat{O}_I(\tau)$ and Heisenberg picture $\hat{O}_H(\tau)$ operators. Relate them by introducing an evolution operator $U(\tau, \tau')$.
- calculate the equation of motion for $U(\tau, \tau')$.
- Write down the solution in integral form for U by direct analogy to the zero temperature case.
- Write down a similar perturbative expansion for the partition function.
- Write the exact temperature Green function in the interaction picture, and arrive at the expression mentioned in class.
- Explicitly write out the zeroth and first order contribution of the numerator.
- Recall the generalized Wick theorem, $\langle T_\tau[\hat{A}\hat{B}\cdots\hat{F}]\rangle$ = sum over all possible contractions (without proof).

- Use Wick's theorem to evaluate the zeroth and first order contribution. Argue that disconnected diagrams drop out against the denominator.
- By comparing with the zero-temperature case, formulate the Feynman rules in momentum space and Matsubara frequency representation.
- Evaluate the first order selfenergy explicitly.
- Draw all second order diagrams for the selfenergy without evaluating them.

Exercise 2: Feynman diagrams for the Hubbard model

Consider the Hubbard model on a simple cubic lattice in standard second quantized notation,

$$H - \mu N = -t \sum_{\langle i,j \rangle \sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\sigma} n_{i-\sigma} - \mu \sum_{i\sigma} n_{i\sigma}. \tag{1}$$

Write down and evaluate the first order selfenergy diagram (the tadpole diagram). By using Dyson's equation, write down a modified Green function G' that sums up all tadpole diagram. In what follows we only look at the *local* modified Green function $G'(\boldsymbol{r} = 0, \omega_n) = \int \frac{d^3k}{(2\pi)^3} G'(\boldsymbol{k}, \omega_n)$ and local selfenergy $\Sigma(\boldsymbol{r} = 0, \omega_n) = \int \frac{d^3k}{(2\pi)^3} \Sigma(\boldsymbol{k}, \omega_n)$. Draw and write down the second order Feynman diagram for the selfenergy in terms of this modified Green function. Note that the expression simplifies in the imaginary time (and coordinate) representation instead of the momentum - Matsubara representation, so you may wish to determine an expression for $G'(\boldsymbol{r} = 0, \tau)$. Finally, draw and label all 3rd and 4th order diagrams for the local self energy (not all Green functions are local now).