6. Problem set

Discussion on Thursday, 9. June 2016

6.1 Exciton binding energy

Exciton binding energy in semiconductors can be calculated analogously to the hydrogen problem if the dielectric constant $\epsilon_r$ and the reduced mass $\mu$ of the respective material are considered. The respective energy spectrum is described by the following formula:

$$E = -\frac{e^4 \mu^2}{4 \pi \epsilon_0 \epsilon_r \hbar^2 n^2}$$

where $e$ is the electron charge, $\mu$ the reduced mass, $\epsilon_0$ the vacuum permittivity and $n$ the principal quantum number.

a) Calculate the binding energy for the hydrogen atom.

Excited exciton states with increasing quantum number $n$ can be observed experimentally (Nature 514, 343-347 (2014)) in absorption spectra following the Rydberg formula $E_n = E_g - E_x/n^2$ with the exciton excited state energy $E_n$, the band-gap energy $E_g$ and the exciton binding energy $E_X$.

b) Determine the band-gap energy of Cu$_2$O ($\epsilon_r = 7.5$), and the exciton binding energy for the so-called yellow P exciton by plotting the photon energy values given for $n = 2$ to $n = 5$ in the cited publication. Determine also the reduced exciton mass $\mu$.

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6.2 Wannier-Mott excitons in GaAs

Consider a Wannier-Mott exciton in GaAs (with $\epsilon_r = 13$, and $m^*_e = 0.067 m_0$ and $m^*_h = 0.34 m_0$ effective electron and heavy hole masses, respectively).

a) Calculate the exciton binding energy. How does it compare to the thermal energy at room temperature and 4 K? Calculate the exciton Bohr radius $a_X$ in nanometers and in units of hydrogen Bohr radius.

b) Calculate the speed of an excitons moving through the lattice with a kinetic energy of 20 meV (i.e. the speed of the center of mass of the bound electron-hole pair). Is it possible for such a moving exciton to recombine radiatively (the band gap at 4 K is $E_g = 1.518$ eV; use also $n = \sqrt{\epsilon_r}$)?