Main idea: contract blocks of tensors = renormalized tensor

In this way, proliferation of free indices is avoided, at the cost of dropping information — as usual in RG schemes!
Details (for square lattice)

Use bipartite partition of lattice:

Fuse vertices and do SVD in two different ways, depending on sublattices:

\[ D \xrightarrow{\text{reshape}} \alpha \xrightarrow{\text{truncate to } D_{\text{cut}}} \beta \xrightarrow{\text{reshape}} \alpha \xrightarrow{\text{SVD}} \mu \]

Fused network

4 x 8 sites = 32 sites

If operators are present, \( \Pi_A, \Pi_B, \) then go along for the ride...

**Computing \( \langle H \rangle \)**

Consider nearest-neighbor term \( \bar{O}^1 \cdot \bar{O}^2 \).

Start with \( \Pi_A = \Pi_1, \Pi_B = \Pi_2, \Pi_C = \Pi_D = \Pi \).

For \( 2^n \) sites, after performing \( n-2 \) step, one ends up with:

For minimum energy, periodic B.C.: connect legs:

\[
\langle 41 \bar{O}^1 \bar{O}^3 | \Psi \rangle = T \left[ \prod_{\Pi \in \Pi_1^{(n-1)}} \prod_{\Pi \in \Pi_2^{(n-1)}} \prod_{\Pi \in \Pi_3^{(n-1)}} \prod_{\Pi \in \Pi_4^{(n-1)}} \right]
\]

\[ \text{cost: } O(D_4 \log N) \]

In this way, compute \( \langle H \rangle \), then optimize elements of original \( \Pi \) variationallly. This is not a linear optimization problem, so not as controlled as the corresponding task in DMRG.
Example: Transverse field Ising model

\[ H = - \sum_{\langle ij \rangle} \hat{\sigma}_i^x \hat{\sigma}_j^x - \frac{h}{2} \sum_i \hat{\sigma}_i^z \]

\[ D = 2, \quad D_{\text{ext}} = 18 \]

critical field:

\[ h_{\text{c}_{\text{TERG}}} = 3.08 \]
\[ h_{\text{c}_{\text{MC}}} = 3.044 \]
\[ h_{\text{c}_{\text{MF}}} = 4 \]

Magnetization:

\[ \langle \sigma_z \rangle = A (|h - h_c|)^\beta \]
\[ \beta_{\text{TERG}} = 0.333 \]
\[ \beta_{\text{MC}} = 0.327 \]
\[ \beta_{\text{MF}} = 0.5 \]

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Correlation functions via TERG

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

Rig step maps shaded square onto black dot. Local operator split onto two sites. Then three. Then four. Not and more than four. shrink until they just touch.