Hydrodynamics SoSe 2019
Problem Sheet 1 (will be discussed May 10, 2019)

Problem 1.1:
1.1.1: Show that the pressure decrease in a planar polytropic atmosphere is described by

$$p(z) = p_0 \left(1 - \frac{n-1}{n}\frac{\rho_0}{p_0} g z\right)^{\frac{n}{n-1}}$$

where g is the gravitational accelaration and ($n \neq 1$) the polytropic index.
What is the expression fort the height h of the atmosphere ($p(h) = 0$)
(Hint: As shown in the lecture, the polytropic equation of state is: $p/\rho^n = const.$)

1.1.2: Show that for n = 1 (isothermal atmosphere), $p(z)$ converges to the barometric formula.

1.1.3: How does atmospheric pressure change above the Earth's surface if we drop the planar assumption and take into account the spherical shape of the Earth? Show that (for $n \neq 1$), the height of the atmosphere converges to that derived in 1.1.1 in the limit $h \ll R$ (R is the Earth's radius).
(Hint: Make use of the potential $\Psi = -G\,M\,/r$, where G is the gravitational constant, M the Earth's mass and r the distance to the centre of the Earth.)

Problem 1.2:
Consider a stationary flow pattern of an ideal gas ($p = nkT$; $n = \rho/m$ is the number density) described by the vector field $\vec{v} = -ay\vec{e}_x + bx\vec{e}_y$ and constant mass density $\rho = \hat{\rho}$ in a homogeneous gravitational field with $\vec{g} = -g\vec{e}_z$. Derive an expression for the corresponding temperature profile.

Problem 1.3:
We consider a stationary flow pattern described by

$$\vec{v} = v_0 \cosh(x)\vec{e}_x + c\vec{e}_y, \qquad v_0, c = const.$$

1.3.1: Draw a sketch of the stream lines y(x).
(Hint: Start by drawing a sketch of the vector field $\vec{c}$ and then draw the stream lines as tangent curves to the vector field. Alternatively, use a computer program.)

1.3.2: Compute the convective derivative of $\vec{v}$.

1.3.3: Determine a non-trivial mass density ρ which satisfies both the given $\vec{v}$ and the stationary equation of continuity.