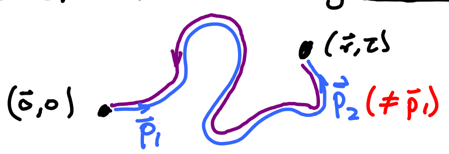
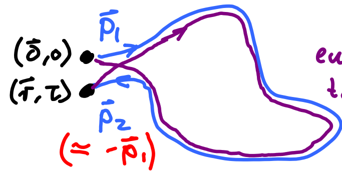
Cooperon: maximally crossed diagrams



$\left[\gg \text{ by } \frac{1}{(k_F \ell)^{d-1}} \right]$

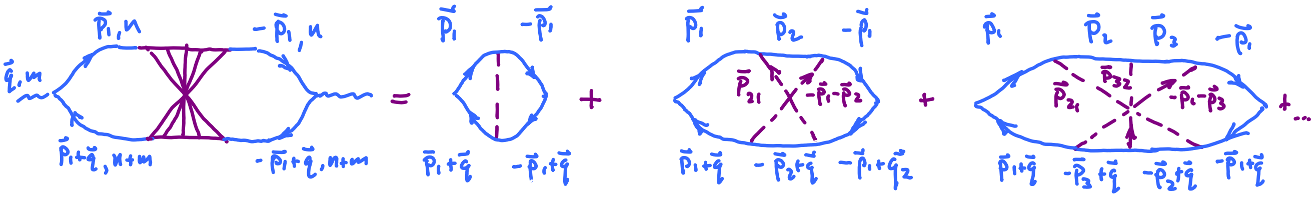


enhances tendency to return to initial point: "weak localization"!

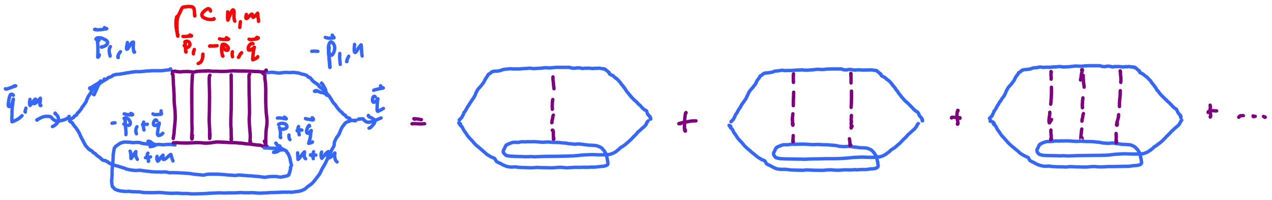
classical contribution: "diffuson"
initial, final momenta uncorrelated: $\vec p_2 \neq \vec p_1$

classical contribution: "Cooperon"
initial, final momenta opposite: $\vec p_2 \simeq -\vec p_1$
this restriction reduces phase space by $\frac{1}{(k_F \ell)^{d-1}}$

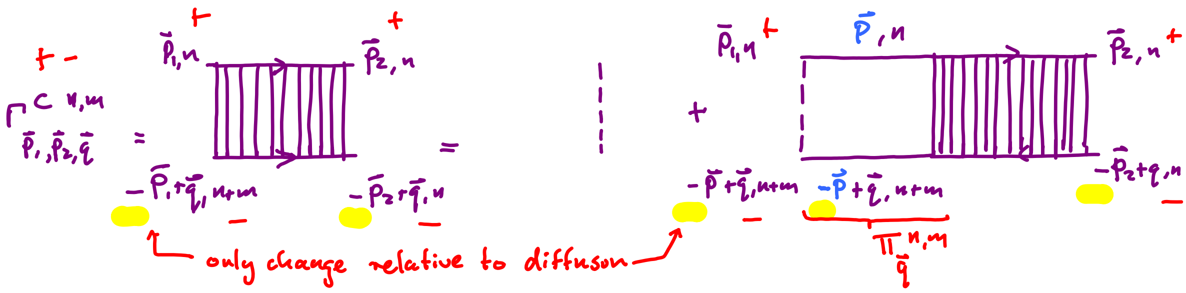
Diagramatically, Cooperon emerges from "maximally crossed diagrams":



= "flip over" lower electron line:



Bethe-Salpeter eq. (BSE) for "Cooperon" vertex in "particle-particle" channel



(1)

$$\text{BSE:} \quad \Gamma^{C,nm}_{\cancel{\vec p_1},\vec p_2,\vec q} = \frac{1}{2\pi\nu_0\tau L^d} + \frac{1}{2\pi\nu_0\tau L^d} \underbrace{\sum_{\vec p} \mathcal{G}_{\vec p,n}\, \mathcal{G}_{-\vec p+\vec q,n+m}}_{\Pi^{n,m}_{\vec q}} \; \Gamma^{C,n,m}_{\cancel{\vec p_1},\vec p_2,\vec q} \Big] \qquad (2)$$

In absence of magnetic field, $\mathcal{G}_{-\vec p+\vec q} = \mathcal{G}_{\vec p-\vec q}$, hence (2) = (13.3) for diffuson (up to $\vec q \to -\vec q$)

Hence $\Pi^{n,m}_{\vec q}$ entering here has same value as for diffuson (for $+-$ choice of ω_n, ω_m; $++$, $--$ give zero)

$$\Rightarrow \text{Cooperon} = \boxed{\Gamma^C_{\vec q,m} \overset{(16.1)}{=} \frac{1}{2\pi\nu_0\tau L^d}\, \frac{1}{|\omega_m| + Dq^2}} \qquad (3)$$

Cooperon in presence of magnetic field "acquires mass"

Switch on magnetic field: $\vec{p} \to (\vec{p} - \vec{A})$ (set $e = c = 1$) (1)

(assume $\vec{A}$ depends only weakly on $\vec{r}$, so $[\vec{p}, \vec{A}] \simeq 0$)

Green's function: $$\mathcal{G}_{\vec{p},n}(\vec{A}) = \left[i\omega_n + E_f - (\vec{p}-\vec{A})^2/2m + \frac{i}{2\tau}\,\mathrm{sgn}(\omega_n) \right]^{-1}$$ (2)

$$\sum_{\vec{p}} \underset{(\vec{p}-\vec{A})^2}{\mathcal{G}_{\vec{p},n}(A)}\ \underset{(-\vec{p}+\vec{q}-\vec{A})^2}{\mathcal{G}_{-\vec{p}+\vec{q}}(A)} \overset{\vec{p}\to\vec{p}+\vec{A}}{=} \sum_{\vec{p}} \underset{(\vec{p})^2}{\mathcal{G}_{\vec{p},n}(0)}\ \underset{(-\vec{p}+\vec{q}-2\vec{A})^2}{\mathcal{G}_{-\vec{p}+\vec{q}}(2\vec{A})} \quad \sum_{\vec{p}} \underset{(\vec{p})^2}{\mathcal{G}_{\vec{p},n}(0)}\ \underset{(-\vec{p}+\vec{q}-2\vec{A})^2}{\mathcal{G}_{-\vec{p}+\vec{q}-2\vec{A}}(0)}$$ (3)

This $\vec{A}$ may be absorbed into redefinition of $\vec{q} \to \vec{q} - 2\vec{A}$

Cooperon in presence of magnetic field $=$ $$\boxed{\Gamma^{C}_{\vec{q},m} \overset{(16.1)}{=} \frac{1}{2\pi\nu_0\tau L^d}\,\frac{1}{|\omega_m| + D(\vec{q}-2\vec{A})^2}}$$ (4)

mass term! divergence for $\omega_m \to 0$, $\vec{q} \to 0$ is cut off. Hence Cooperon decays exp. with time

More generally, if $\vec{A}$ depends on $\vec{r}$ (smoothly on scale of ℓ), then Γ is formally defined as solution of: $$\left(|\omega_m| + D(-i\vec{\nabla}_{\vec{r}} - 2\vec{A})^2 \right) \Gamma(\vec{r},\omega_n) = \frac{1}{2\pi\nu_0 L^d}\,\delta(\vec{r})$$ (5)