LMU München
WS 2016/2017
Lehrstuhl für Theoretische Nanophysik
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13. Exercise Sheet TA1: Theoretical Solid State Physics

To be discussed on Tuesday, February 7, 2017.

Exercise 1: Two-site Hubbard model

Consider the Hubbard dimer, which in second quantization reads:

$$H = -t \sum_{\sigma=\uparrow,\downarrow} (c^\dagger_{1\sigma} c_{2\sigma} + h.c.) + U \sum_{i=1,2} n_{i\uparrow} n_{i\downarrow} \,.$$

$c_{i\sigma}$ annihilates an electron with spin σ on site i and $n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma}$.

a. For $U = 0$, $t > 0$, show that the Hamiltonian can be written as

$$H = \sum_{\mu,\sigma} \epsilon_\mu c^\dagger_{\mu\sigma} c_{\mu\sigma}$$

Express the new operators $c^\dagger_{\mu\sigma}$ and $c_{\mu\sigma}$ through the original ones. Compute the ground-state energy for $U = 0$ and two particles with opposite spin.

b. Now consider the interacting case $U > 0$. Calculate $E_{\text{var}} = \langle\psi_0|H|\psi_0\rangle$ where $|\psi_0\rangle$ is the two-particle ground state of the non-interacting case for two electrons with opposite spin. Interpret all terms that appear in this expression. What is the probability for two electrons to be on the same site for the wave-function $|\psi_0\rangle$?

Exercise 2: Phonons: Density of states

Consider phonons on a 3D Bravais lattice. Show that the density of normal modes behaves as

$$g(\omega) \propto \sqrt{\omega_0 - \omega}$$

in the vicinity of a maximum of the phonon dispersion where ω_0 is the value of $\omega(\boldsymbol{k})$ at the maximum. Assume that $\omega(\boldsymbol{k}) = \omega(k)$, $k = |\boldsymbol{k}|$.

Exercise 3: Thermopower

Compute the leading temperature dependence of the thermopower $Q = L^{12}/L^{11}$ of a metal.

Exercise 4: Thomas-Fermi approximation

Derive the Thomas-Fermi form of the dielectric constant of a metal starting from the Lindhard function.