LMU München
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Lehrstuhl für Theoretische Nanophysik
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14. Exercise Sheet TA1: Theoretical Solid State Physics

To be discussed on Friday, February 10, 2017.

Exercise 1: Specific heat of a spin system in a magnetic field

We consider a system of N uncoupled spin-$\frac{1}{2}$ moments in an external magnetic field B. The Hamiltonian is given by

$$H = \boldsymbol{B} \cdot \sum_j \boldsymbol{S}_j$$

a. Compute the specific heat C_V.

b. Discuss the high-temperature limit.

Exercise 2: Anisotropic Heisenberg Model

Consider the anisotropic Heisenberg spin-S Hamiltonian

$$H = -\frac{1}{2} \sum_{\boldsymbol{R},\boldsymbol{R}'} [J_z(\boldsymbol{R}-\boldsymbol{R}') S^z(\boldsymbol{R}) S^z(\boldsymbol{R}') + \frac{J(\boldsymbol{R}-\boldsymbol{R}')}{2} (S^+(\boldsymbol{R}) S^-(\boldsymbol{R}') + S^-(\boldsymbol{R}) S^+(\boldsymbol{R}'))],$$

with $J_z(\boldsymbol{R}-\boldsymbol{R}') > J(\boldsymbol{R}-\boldsymbol{R}') > 0$.

a. Show that the groundstate of the isotropic ferromagnet is

$$|0\rangle = \prod_{\boldsymbol{R}} |S\rangle_{\boldsymbol{R}},$$

and that the one-spin-wave states

$$|\boldsymbol{k}\rangle = \frac{1}{\sqrt{N}} \sum_{\boldsymbol{R}} \mathrm{e}^{i\boldsymbol{k}\boldsymbol{R}} |R\rangle, \text{ where } |\boldsymbol{R}\rangle = \frac{1}{\sqrt{2S}} S^-(\boldsymbol{R}) |0\rangle,$$

remain eigenstates of H, but that the spin-wave excitation energies are raised by

$$S \sum_{\boldsymbol{R}} [J_z(\boldsymbol{R}) - J(\boldsymbol{R})],$$

compared to the isotropic case.

b. Show that the low-temperature spontaneous magnetization now deviates from saturation only exponentially in $-\frac{1}{T}$.

c. Show that the argument for the absence of spontaneous magnetization in 2D given in the lecture no longer holds.

Exercise 3: Magnetic susceptibility, critical exponents

The free energy function F of a ferromagnetic Ising model close to the transition temperature T_c in mean-field theory is given by

$$F(m, h) = F_0 + am^2 + bm^4 - hm$$

where $F_0 = \text{const}$, $b > 0$, $a(T) = \alpha(T - T_c)$, $\alpha > 0$, h is the magnetic field, and m is the order parameter. Compute the temperature dependence of the zero-field magnetic susceptibility χ close to T_c.