LMU München
WS 2014/2015
Lehrstuhl für Theoretische Nanophysik
PD Dr. F. Heidrich-Meisner
Jan Stolpp, *Questions: jan.stolpp@physik.uni-muenchen.de*

6. Exercise Sheet TA1: Theoretical Solid State Physics

To be discussed on Friday, December 2, 2016.

Exercise 1: van Hove singuarities

In a linear harmonic chain with only nearest-neighbour interactions, the normal mode dispersion relation has the form $\omega(k) = \omega_0 |\sin(ka/2)|$, where ω_0 is the maximum frequency.

a. Calculate the density of normal modes

b. Discuss what happens at $\omega = \omega_0$.

Exercise 2: Three-phonon processes in one dimension

Consider a process in which two phonons combine to give a third (or one phonon decays into two others). Let all phonons be acoustic, assume that the two transverse branches lie below the longitudinal branch. Assume that all wavevectors lie along a fixed direction and assume that $d^2\omega/dk^2 \leq 0$ for all three branches.

a. Interpreting the conservation laws graphically, show that there can be no process in which all three phonons belong to the same branch.

b. Show that the only possible processes are those in which the single phonon is in a branch higher than at least one of the members of the pair; i.e.,

$$\text{transverse} + \text{transverse} \leftrightarrow \text{longitudinal}$$

or

$$\text{transverse} + \text{longitudinal} \leftrightarrow \text{longitudinal}.$$

Exercise 3: Anharmonic potential

Sketch all phonon scattering processes that can occur due to quartic terms of the anharmonic potential.

Exercise 4: Low temperature specific heat in d-dimensions, and for nonlinear dispersion laws

a. Show that the density of normal modes for the Debye approximation,

$$g_D(\omega) = \frac{3}{2\pi^2}\frac{\omega^2}{c^3} \qquad \text{if}\, \omega < \omega_D = k_D c \tag{1}$$
$$0 \qquad \text{if}\, \omega > \omega_D = k_D c \tag{2}$$

gives the exact (within the harmonic approximation) leading *low-frequency* behaviour of $g(\omega)$, provided that the velocity c is defined by

$$\frac{1}{c^3} = \frac{1}{3}\sum_s \int \frac{d\Omega}{4\pi}\frac{1}{c_s^3(\hat{\boldsymbol{k}})} \,.$$

b. Show that in a d-dimensional harmonic crystal, the low-frequency density of normal modes varies as ω^{d-1}.

c. Deduce from this that the low-temperature specific heat of a harmonic crystal vanishes as T^d in d dimensions.

d. Show that, if it should happen that the normal mode frequencies did not vanish linearly with k, but as k^ν, then the low-temperature specific heat would vanish as $T^{d/\nu}$, in d dimensions.