LMU München
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Lehrstuhl für Theoretische Nanophysik
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7. Exercise Sheet TA1: Theoretical Solid State Physics

To be discussed on Friday, December 9, 2016.

Exercise 1: Bloch electrons in a uniform magnetic field

The motion of Bloch electrons in a uniform magnetic field is determined through the semi-classical equations of motion:

$$\dot{\boldsymbol{r}} = \boldsymbol{v}(\boldsymbol{k}) = \frac{1}{\hbar}\frac{\partial \epsilon(\boldsymbol{k})}{\partial \boldsymbol{k}}, \tag{1}$$

$$\hbar\dot{\boldsymbol{k}} = -\frac{e}{c}\boldsymbol{v}(\boldsymbol{k}) \times \boldsymbol{H}. \tag{2}$$

Derive the real-space orbits of the electrons projected onto a plane perpendicular to the magneitc field in terms of the k-space orbits.

Exercise 2: Continuity equation

Using the Boltzmann equation, derive the continuity equation:

$$\nabla_{\boldsymbol{r}} \boldsymbol{j} = -\frac{\partial \rho}{\partial t}\,. \tag{3}$$

Exercise 3: Kinetic equation – non-linear I-V characteristics

Consider one-dimensional Bloch electrons. The simplest dispersion relation in the tight-binding approximation reads:

$$\epsilon(p) = \Delta\left[1 - \cos\left(\frac{pd}{\hbar}\right)\right] \tag{4}$$

where Δ and d are the band width and the lattice constant respectively. A constant electric field E_0 is applied.

a. Write down the classical kinetic equation, and use the momentum independent relaxation time approximation for the collision integral.

b. Solve the kinetic equation for the distribution function $g(p,t)$.

c. Compute the current density :

$$j = \frac{2e}{h} \int dp\, v(p) g(p,t)\,, \tag{5}$$

where $v(p)$ is the electron velocity.

d. Analyze the dependence of j on the electric field E_0 in the case of strong and weak field and identify the crossover from the linear to the non-linear regime.