LMU München
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Lehrstuhl für Theoretische Nanophysik
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8. Exercise Sheet TA1: Theoretical Solid State Physics

To be discussed on Tuesday, December 13, 2016.

Exercise 1: Conductivity tensor of a crystal with cubic symmetry

Argue that the conductivity tensor of a crystal with cubic symmetry is

$$\sigma_{\mu\nu} = \sigma_0 \delta_{\mu\nu} \,. \tag{1}$$

Hint: Remember that $\boldsymbol{j} = \boldsymbol{\sigma} \boldsymbol{E}$, where $\boldsymbol{\sigma}$ is the conductivity tensor and $\boldsymbol{E}$ is the electric field.

Exercise 2: Electrical conductivity

a. Study the expression for the DC conductivity given in Chap. 6.8 of the lecture. Under which assumption does this reduce to the Drude formula ?

b. Use the result for the non-equilibrium distribution function $g(\boldsymbol{k}, \boldsymbol{r}, t)$ derived from the relaxation time approximation to derive the formula for the AC conductivity, in the case of a uniform temperature and chemical potential :

$$\sigma^{(n)}_{\mu\nu}(\omega) = e^2 \int \frac{d^3\boldsymbol{k}}{4\pi^3} \frac{v^{(n)}_\mu(\boldsymbol{k})\, v^{(n)}_\nu(\boldsymbol{k})}{1 - i\omega\tau^{(n)}(\epsilon(\boldsymbol{k}))} \tau^{(n)}\left[\epsilon(\boldsymbol{k})\right] \left(-\frac{\partial f}{\partial \epsilon}\right)_{\epsilon=\epsilon^{(n)}(\boldsymbol{k})} . \tag{2}$$

c. Show that the AC conductivity, in the limit of $\omega\tau \ll 1$, reduces to:

$$\sigma^{(n)}_{\mu\nu}(\omega) = -\frac{e^2}{i\omega}\frac{1}{\hbar^2} \int \frac{d^3\boldsymbol{k}}{4\pi^3} f\left[\epsilon^{(n)}(\boldsymbol{k})\right] \frac{\partial^2 \epsilon^{(n)}(\boldsymbol{k})}{\partial k_\mu k_\nu} \tag{3}$$

Hint: Use an electric field that has the form $\boldsymbol{E}(t) = \mathrm{Re}\left[\boldsymbol{E}(\omega) e^{-i\omega t}\right]$.

Exercise 3: Transport coefficients

The transport coefficient in the isotropic case L^{ij} are defined via:

$$\boldsymbol{j} = L^{11}\mathcal{E} + L^{12}(-\nabla T), \tag{4}$$
$$\boldsymbol{j}^q = L^{21}\mathcal{E} + L^{22}(-\nabla T), \tag{5}$$

connecting the electrical current density $\boldsymbol{j}$ and the thermal current density $\boldsymbol{j}^q$ to the external forces $\mathcal{E} = \boldsymbol{E} + \nabla\mu/e$ and ∇T.

a. Argue that for a typical metal, the 2nd contribution to the thermal conductivity stemming from the off-diagonal coefficients can be ignored.
Hint: compare L^{22} *to* $L^{21}{L^{11}}^{-1}L^{12}$.

b. In order to apply the Sommerfeld expansion for the calculation of the L^{ij} in the lecture, we have carried out an integration by parts and dropped surface terms. Why is this correct?

c. Derive the expression for L^{22} given in the lecture.

d. Calculate the thermopower $Q = L^{12}/L^{11}$ and explain the Seebeck and Peltier effects.
Hint: you can read about these effects in standard textbooks.