Exercises on General Relativity TVI TMP-TC1

Problem set 13, due 29.01 - 03.02

Exercise 1 – Einstein tensor for weak field

Consider the weak field approximation (linearized gravity) and regard the metric as a small perturbation h around Minkowski space η with $g = \eta + h$.

(i) Find the linearized Einstein tensor neglecting all terms $O(h^2)$

$$G^{(1)}_{\mu\nu} = \frac{1}{2}\left(\partial_\mu \partial^\lambda \bar{h}_{\lambda\nu} + \partial_\nu \partial^\lambda \bar{h}_{\lambda\mu} - \eta_{\mu\nu}\partial^\lambda\partial^\rho \bar{h}_{\lambda\rho} - \Box \bar{h}_{\mu\nu}\right) \tag{1}$$

with $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ and $h = h_\mu{}^\mu$.

(ii) Recall the harmonic gauge with $g^{\mu\nu}\Gamma^\alpha_{\mu\nu} = 0$ and show that the linearized Einstein tensor in this gauge is given by

$$G^{(1)}_{\mu\nu} = -\frac{1}{2}\Box \bar{h}_{\mu\nu}\ . \tag{2}$$

(iii) Compare the linearized Einstein tensor with the equation of motion of the Fierz-Pauli action.

Exercise 2 – Geodesics in the space-time of the sun

Approximate the space-time outside of the sun with mass M as

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -f(r)\,\mathrm{d}t^2 + \frac{1}{f(r)}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2 \tag{3}$$

with $f(r) := \left(1 - \frac{2GM}{r}\right)$ and $\mathrm{d}\Omega = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2$.

(i) Convince yourself that the limit $r \to \infty$ is correct.

(ii) Write down the Lagrangian of a massive point particle moving in this space-time.

(iii) Exploit the explicit independence of the metric on t and ϕ to find the corresponding constants of motion E and L.

From now on restrict to the equatorial plane $\theta = \frac{\pi}{2}$.

(iv) Find a relation for E^2 using $u^\mu u_\mu = -1$ with the four velocity u.

(v) Subtracting the rest energy and the radial kinetic energy, find the effective potential for this particle.

(vi) Sketch the effective potential for $L \neq 0$, discuss possible geodesics and take the non-relativistic limit. From these results how can you decide whether you have to use General Relativity or Newton's gravity to describe the solar system?

Exercise 3 – Bending of light

Verify that the gravitational bending of light passing near the sun is

$$\delta = 1.75'' \frac{R_\odot}{R} \tag{4}$$

where R is the distance at which the light passes from the center of the sun and $R_\odot$ is the radius of the sun.

General information

The lecture takes place on Monday at 14:00-16:00 and on Friday at 10:00 - 12:00 in A348 (Theresienstraße 37).

Presentation of solutions:
Monday at 16:00 - 18:00 in B 138

There are six tutorials:
Monday at 12:00 - 14:00 in A 249
Thursday at 16:00 - 18:00 in A 449
Friday at 14:00 - 16:00 in C 113 and A 249
Friday at 16:00 - 18:00 in A 249

The webpage for the lecture and exercises can be found at

`www.physik.uni-muenchen.de/lehre/vorlesungen/wise_17_18/tvi_tc1_gr/index.html`