

Exercises on General Relativity TVI TMP-TC1

Problem set 6, due December 4th

Exercise 1 – Charge carrier on a table

Consider a charge carrier placed on a table in a flat spacetime. What are the conditions on the charge such that it radiates electromagnetic waves? What changes when you place this setup in a curved spacetime and let it fall freely? Imagine a power unit which prevents the table to fall and reconsider the situation.

Exercise 2 – Charts on the 2-sphere

Consider the two dimensional sphere S^2 and its realization in \mathbb{R}^3 as:

$$\sum_{j=0}^2 (x^j)^2 = 1 \quad (1)$$

Define coordinate neighbourhoods as follows

$$U_{j+} := \{(x^0, x^1, x^2) \in S^3 | x^j > 0\} \quad (2)$$

$$U_{j-} := \{(x^0, x^1, x^2) \in S^3 | x^j < 0\} \quad (3)$$

On each of the neighbourhoods $U_{i\pm}$ define a projection map, projecting out one of the coordinates, for example:

$$\phi : U_{0\pm} \longrightarrow \mathbb{R}^2 \quad (4)$$

$$\phi_{0\pm}(x^0, x^1, x^2) = (x^1, x^2) \quad (5)$$

and similarly for all other $\phi_{i\pm}$ and x^i respectively. Calculate the transition functions $\phi_{1-} \circ \phi_{2+}^{-1}$ and $\phi_{2+} \circ \phi_{1-}^{-1}$ on the respective domains.

Exercise 3 – Fun with Diffeomorphisms

(i) Show that the assignment given by

$$x \mapsto \frac{ax}{\sqrt{a^2 - x^2}} \quad (6)$$

defines a diffeomorphism between $(-a, a)$ and \mathbb{R} with a positive real number a .

(ii) Define a diffeomorphism between the open ball

$$B_a := \{x \in \mathbb{R}^n \mid \|x\| < a\} \quad (7)$$

and the entire \mathbb{R}^n .

(iii) Find a smooth function $f : \mathbb{R}^n \rightarrow \mathbb{R}_0^+$ with the following properties:

$$f(x) = 1 \quad x \in \bar{B}_1 \quad (8)$$

$$f(x) = 0 \quad x \in \mathbb{R}^n \setminus B_2, \quad (9)$$

where \bar{B} denotes a closed ball.

General information

The lecture takes place on Monday at 14:00-16:00 and on Friday at 10:00 - 12:00 in A348 (Theresienstraße 37).

Presentation of solutions:

Monday at 16:00 - 18:00 in B 138

There are six tutorials:

Monday at 12:00 - 14:00 in A 249

Thursday at 16:00 - 18:00 in A 449

Friday at 14:00 - 16:00 in C 113 and A 249

Friday at 16:00 - 18:00 in A 249

The webpage for the lecture and exercises can be found at

www.physik.uni-muenchen.de/lehre/vorlesungen/wise_17_18/tvi_tc1_gr/index.html