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11th Exercise Sheet Many-Body Physics

Will be discussed in the week of July 8-12.

Exercise 1: BCS pairing amplitudes FW 10.7

Compute the pairing amplitudes $F_k^* \equiv \langle \mathbf{0}|a^\dagger_{\boldsymbol{k},\uparrow}a^\dagger_{-\boldsymbol{k}\downarrow}|\mathbf{0}\rangle$ and $F_k \equiv \langle \mathbf{0}|a_{\boldsymbol{k},\uparrow}a_{-\boldsymbol{k}\downarrow}|\mathbf{0}\rangle$ in the BCS ground state. Sketch their behavior as a function of k, and show that they vanish in the normal ground state.

Exercise 2: BCS wave function FW 10.8

The superconducting ground state was orginally derived with a variational principle by considering the state

$$|\varphi\rangle = \prod_{\boldsymbol{k}} (u_k + v_k a^\dagger_{\boldsymbol{k},\uparrow}a^\dagger_{-\boldsymbol{k}\downarrow})|0\rangle, \tag{1}$$

where the product is over all $\boldsymbol{k}$ and $|0\rangle$ is the no-particle state.

a. Show that $|\varphi\rangle$ is normalized if $u_k^2 + v_k^2 = 1$.

b. Compute the expectation value of the system

$$\begin{aligned} K &= H - \mu N \\ &= \sum_{\boldsymbol{k}\lambda} a^\dagger_{\boldsymbol{k}\lambda} a_{\boldsymbol{k}\lambda}(\epsilon_k^0 - \mu) - \frac{1}{2} \sum_{\boldsymbol{k_1}+\boldsymbol{k_2}=\boldsymbol{k_3}+\boldsymbol{k_4}} \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} [\ldots] \\ [\ldots] &= \langle \boldsymbol{k_1}\lambda_1\boldsymbol{k_2}\lambda_2|V|\boldsymbol{k_3}\boldsymbol{k_4}\rangle a^\dagger_{\boldsymbol{k_1}\lambda_1} a^\dagger_{\boldsymbol{k_2}\lambda_2} a_{\boldsymbol{k_4}\lambda_4} a^\dagger_{\boldsymbol{k_3}\lambda_3} \end{aligned} \tag{2}$$

c. Varying u_k and v_k subject to the constraint $u_k^2 + v_k^2 = 1$, derive the gap equation as the condition for minimum thermodynamic potential.

d. Apart from normalization, verify that $a^\dagger_{\boldsymbol{k}\uparrow}|\varphi\rangle$ and $a_{-\boldsymbol{k}\downarrow}|\varphi\rangle$ both represent the same state which is orthogonal to $|\varphi\rangle$. Evaluate the expectation value of K in this state and show that the increase in the the thermodynamic potential is E_k.

Exercise 3: BCS density of states

Compute the local density of states for a BCS superconductor, $\rho(\epsilon) = \sum_{k,\text{branch}}(\epsilon - E_k)$, where E_k is the Bogoliubov quasi-particle energy.