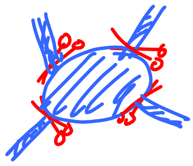
$m=2$ vertex functions



Goal: to show that

$$\gamma_2(k_1',k_2';k_1,k_2) \overset{(I.20.3)}{=} -\sum_{q_1',q_2',q_1,q_2} [\mathcal{G}]^{-1}_{k_1'q_1'}\,[\mathcal{G}^{-1}]_{k_2'q_2'}\,\mathcal{G}_2^c[q_1',q_2';q_1,q_2]\,[\mathcal{G}^{-1}]_{q_1k_1}\,[\mathcal{G}^{-1}]_{q_2k_2} \qquad (1)$$

Starting point:

$$\delta_{kk'} \overset{(I.14.5)}{=} \sum_q \left\{ \frac{\delta^2\Gamma}{\delta\phi_{k'}\delta\bar\phi_q} - [\mathcal{G}_0^{-1}]_{qk'} \right\} \frac{\delta^2 W^c}{\delta\eta_q\delta\bar\eta_k} + \zeta\, \cancel{\frac{\delta^2\Gamma}{\delta\phi_{k'}\delta\phi_q}}\, \underbrace{\frac{\delta^2 W^c}{\delta\bar\eta_q\delta\bar\eta_k}} \Bigg\} \qquad (2)$$

will yield zero when setting $\phi=\bar\phi=0$ at the end.

Take $\dfrac{\delta^2}{\delta\bar\phi_{\ell'}\delta\phi_\ell}$ (2):

$$0 = \sum_q \Bigg[\underbrace{\left(\frac{\delta^2}{\delta\bar\phi_{\ell'}\delta\phi_\ell}\frac{\delta^2\Gamma}{\delta\phi_{k'}\delta\bar\phi_q}\right)}_{(I.13.1)} \underbrace{\frac{\delta^2 W^c}{\delta\eta_q\delta\bar\eta_k}}_{(I.6.5)} + \underbrace{\left\{\frac{\delta^2\Gamma}{\delta\phi_{k'}\delta\bar\phi_q} - [\mathcal{G}_0^{-1}]_{qk'}\right\}}_{(I.14.5)} \sum_{s's} \underbrace{\frac{\delta\bar\eta_{s'}}{\delta\bar\phi_{\ell'}}}_{(2.2)} \underbrace{\frac{\delta\eta_s}{\delta\phi_\ell}}_{(2.1)} \underbrace{\frac{\delta^2}{\delta\bar\eta_{s'}\delta\eta_s}\frac{\delta^2 W^c}{\delta\eta_q\delta\bar\eta_k}}_{(I.13.1)} \Bigg]_{\phi=0=\bar\phi} \qquad (3)$$

$$\sum_q \gamma_2(\ell',q;k',\ell)\,\zeta\, G_1^c(k;q) + \sum_{ss'}\sum_q \zeta\,(G_1^c)^{-1}_{qk'}\,(-\zeta G_1^c)^{-1}_{\ell's'}\,(-\zeta G_1^c)^{-1}_{s\ell}\; G_2^c(s',k;q,s) \qquad (4)$$

For (1.4), we used:

$$\boxed{\begin{aligned} \frac{\delta\eta_s}{\delta\phi_\ell} &\overset{(I.16.3)}{=} -\frac{\delta^2\Gamma}{\delta\phi_\ell\delta\bar\phi_s} + [\mathcal{G}_0^{-1}]_{s\ell} = -\zeta\,[G_1^c]^{-1}_{s\ell} && (1)\\ \frac{\delta\bar\eta_{s'}}{\delta\bar\phi_{\ell'}} &\overset{(I.15.4)}{=} -\zeta\frac{\delta^2\Gamma}{\delta\bar\phi_{\ell'}\delta\phi_{s'}} + [\mathcal{G}_0^{-1}]_{\ell' q} = -\zeta\,[G_1^c]^{-1}_{\ell's'} && (2) \end{aligned}}$$

This is how change of variables from η to ϕ produces full Green's functions!!

$\sum (G_1^c)^{-1}_{\bar k k}$ (1.4)

$$\gamma_2(\ell',\bar k;k',\ell) = -\sum_{kss'q} (G_1^c)^{-1}_{\bar k k}\,(G_1^c)^{-1}_{qk'}\,(G_1^c)^{-1}_{\ell's'}\,(G_1^c)^{-1}_{s\ell}\; G_2^c(s',k;q,s) \qquad (3)$$

relabel: $\ell'\to k_1'$; $\bar k\to k_2'$; $k'\to k_1$; $\ell\to k_2$; $s'\to q_1'$; $k\to q_2'$; $q\to q_1$; $s\to q_2$

$$\boxed{\gamma_2(k_1',k_2';k_1,k_2) = \sum_{q_1'q_2'q_1q_2} (G_1^c)^{-1}_{k_1'q_1'}\,(G_1^c)^{-1}_{k_2'q_2'}\; G_2^c(q_1',q_2';q_1,q_2)\,(G_1^c)^{-1}_{q_1k_1}\,(G_1^c)^{-1}_{q_2k_2}} \qquad (4)$$

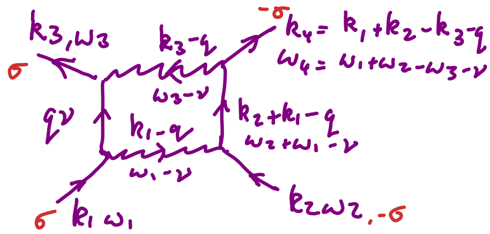
$= (1.1)$ ✓

Infrared problems in pert. theory

Perturbation theory for low-dimensional systems often produces infrared divergencies.

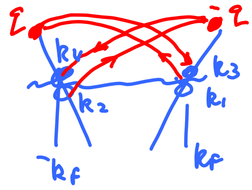
Example: Consider interacting electron gas in 1D:

A diagram with log. divergence is:



Consider for simplicity: $k_1 = k_3 = k_F$, $k_2 = k_4 = -k_F$

$$F(\omega_1+\omega_2) = \frac{1}{\beta L} \sum_{q\nu} \tilde{v}^2(k_F - q) \frac{1}{i\nu - \xi_q} \frac{1}{i(\omega_1+\omega_2-\nu) - \xi_{-q}} \qquad (1)$$



One dominant contribution comes when $q \approx -k_F + q'$, q' small

so that $\xi_{\pm q} = \varepsilon_{\pm q} - \varepsilon_F = -v_F(\pm q + k_F) = \mp v_F q'$ (linearize, for $|q'| < \Lambda_0$) cutoff. (2)

Then, for $T \to 0$:

$$F(\omega_1+\omega_2) = \frac{i}{2\pi} \tilde{v}^2(k_F) \left[\int_0^{\Lambda_0} dq - \int_{-\Lambda_0}^{0} dq \right] \frac{1}{\omega_1+\omega_2 - i2v_F q} \sim \ln\left| \frac{\Lambda_0}{\omega_1+\omega_2} \right| \qquad (3)$$

This diverges for $\omega_1 + \omega_2 \to 0$.

General strategy: To avoid infrared divergencies, modify propagators:

replace $\mathcal{G}^0_q(i\omega_n) \longrightarrow \mathcal{G}^{0,\Lambda}_q(i\omega_n) = \Theta(|\xi_q| - \Lambda)\, \mathcal{G}^0_q(i\omega_n)$ (1a)

or $\mathcal{G}^0_q(i\omega_n) \longrightarrow \mathcal{G}^{0,\Lambda}_q(i\omega_n) = \Theta(|\omega_n| - \Lambda)\, \mathcal{G}^0_q(i\omega_n)$ (1b)

(specific choice can be fixed later). Then, all vertex functions will depend on Λ, so

$$\frac{d}{d\Lambda} \gamma_m^\Lambda = \mathcal{F}(\gamma_1^\Lambda, \dots, \gamma_m^\Lambda; \Lambda) \qquad (= \text{some function of } \Lambda) \qquad (2)$$

At $\Lambda = \infty$, $\mathcal{G}^{0,\Lambda} = 0$, hence $\gamma_m = \begin{cases} 0 & \text{for } m \neq 2 \\ v_{\text{bare}} & \text{for } m = 2 \end{cases}$ (3)

Integrate (2): $\gamma_m^{\Lambda=0} = \int_\infty^0 d\Lambda\, \mathcal{F}(\gamma_1^\Lambda, \dots, \gamma_m^\Lambda; \Lambda) - \delta_{m2} v_{\text{bare}}$

yields desired vertex functions of theory with full ($\Lambda = 0$) propagators.

Advantage of this way of calculating $\gamma_m^{\Lambda=0}$: along the way, all infrared problems throughout are regularized by Λ. So, there is hope that $\int_\infty^0 d\Lambda \dots$ also remains well-behaved.

Eg: $\lim_{\Lambda \to 0} \ln\left|\frac{\Lambda}{\omega_1}\right| \to \infty$, but $\int_{\Lambda_0}^0 d\Lambda \ln\left|\frac{\Lambda}{\omega_1}\right| \neq \infty$

Flow equation for generating Functionals

After replacement $\mathcal{G}^0 \to \mathcal{G}^{0,\Lambda}$ all generating functionals depend on Λ:

Eq. $$\frac{d}{d\Lambda}\Gamma = \tilde{\mathcal{F}}(\Gamma, \phi, \bar\phi, \Lambda, \mathcal{G}^0, \dots) \qquad (1)$$

"Expanding" both sides of (1) in powers of $\phi\bar\phi$, gives flow eqs for γ_m of the form (4.2).

Define:

$$W^\Lambda[\eta,\bar\eta] = \int \mathcal{D}(\bar\psi,\psi)\, e^{(\bar\psi,\, \mathcal{G}^{0,\Lambda}\psi) - S_{int}(\psi,\bar\psi) - (\bar\psi,\eta^\Lambda) - (\bar\eta^\Lambda,\psi)} \qquad (2)$$

$$Z_0^\Lambda = \int \mathcal{D}(\bar\psi,\psi)\, e^{(\bar\psi,\, \mathcal{G}^{0,\Lambda}\psi)} \qquad (3)$$

$$W^{c,\Lambda}[\eta,\bar\eta] = \ln\left(W^\Lambda / Z_0^\Lambda\right) \qquad (4)$$

[η^Λ and $\bar\eta^\Lambda$ have to be chosen Λ-dependent, via (I.12.3,6), in such a way that $\phi, \bar\phi$, which are the natural variables of $\Gamma[\phi,\bar\phi]$, are Λ-independent.]

Note that we normalize (2) by Z_0^Λ, not Z^Λ. (5)

$$\mathcal{G}_0^{c,\Lambda} = W^{c,\Lambda}\big|_{\eta=0=\bar\eta} = \ln\left(\frac{\overbrace{W^\Lambda[0,0]}^{=Z^\Lambda}}{Z_0^\Lambda}\right) \neq 0 \qquad (6) \qquad \text{(in contrast to I-6.6)}$$

$$\overset{\Lambda=0}{\Longrightarrow} \quad \ln Z = \underbrace{\mathcal{G}_0^{c,\Lambda=0}}_{-\gamma_0^{\Lambda=0}} - \ln Z_0^{\Lambda=0} \qquad (7)$$

$\Longrightarrow$ Z can be obtained from $\gamma_0^{\Lambda=0}$

[$m>0$ correlators are unaffected by (4)].

Now calculate $\frac{d}{d\Lambda}$; tedious, but straightforward:

$$\dot W^{c,\Lambda} \equiv \frac{dW^{c,\Lambda}}{d\Lambda} \overset{(5.4)}{=} \frac{d}{d\Lambda}\ln\left(W^\Lambda / Z_0^\Lambda\right) \qquad (1)$$

To simplify notation, write $C^\Lambda = (\mathcal{G}^{0,\Lambda})^{-1}$, e.g.

$$W^\Lambda = \int \mathcal{D}(\bar\psi,\psi)\, e^{(\bar\psi, C^\Lambda\psi) - S_{int} - (\bar\eta^\Lambda,\psi) - (\bar\psi,\eta^\Lambda)} \qquad (2)$$

$$Z_0^\Lambda = \int \mathcal{D}(\bar\psi,\psi)\, e^{(\bar\psi, C^\Lambda\psi)} \qquad (3)$$

Then

$$\dot W^{c,\Lambda} = \Big(\dot{\bar\eta}^\Lambda, \overbrace{\frac{\delta W^{c,\Lambda}}{\delta\bar\eta^\Lambda}}^{-\phi}\Big) \overset{(I.11.1)}{+} \Big(\dot\eta^\Lambda, \overbrace{\frac{\delta W^{c,\Lambda}}{\delta\eta^\Lambda}}^{-S\bar\phi}\Big) + \frac{1}{W^\Lambda}\dot C^\Lambda \frac{\delta W^\Lambda}{\delta C^\Lambda} - \frac{1}{Z_0^\Lambda}\dot C^\Lambda\frac{\delta Z_0^\Lambda}{\delta C^\Lambda} \qquad (4)$$

$$\frac{1}{Z_0^\Lambda}\dot C^\Lambda \frac{\delta Z_0^\Lambda}{\delta C^\Lambda} = \frac{1}{Z_0^\Lambda}\int\mathcal{D}(\bar\psi,\psi)\, \overbrace{(\bar\psi, \dot C^\Lambda\psi)}^{S}\, e^{(\bar\psi, C^\Lambda\psi)} \overset{\langle\bar\psi\psi\rangle_0 = \mathcal{G}^0 \ (6.1)}{=} \mathrm{Tr}\left[\dot C^\Lambda \mathcal{G}^{0,\Lambda}\right] \qquad (5)$$

$\left[\sum_{kk'} \dot C^\Lambda_{kk'} (\mathcal{G}^{0,\Lambda})_{k'k}\right]$

Similarly:

$$\frac{1}{W^\Lambda}\dot{C}^\Lambda \frac{\delta W^\Lambda}{\delta C^\Lambda} = \frac{1}{W^\Lambda}\int \mathcal{D}(\bar\psi,\psi)\,(\bar\psi, \dot C^\Lambda\psi)\, e^{(\bar\psi, C^\Lambda \psi) - S_{int} - (\bar\eta^\Lambda,\psi) - (\bar\psi,\eta^\Lambda)} \tag{1}$$

$$= \frac{1}{W^\Lambda}\, \mathrm{Tr}\left[\dot C^\Lambda \frac{\delta}{\delta\bar\eta^\Lambda}\underbrace{\frac{\delta}{\delta\eta^\Lambda}\; W^\Lambda}\right] \tag{2}$$

$$= \frac{1}{W^\Lambda}\, \mathrm{Tr}\left[\dot C^\Lambda \frac{\delta}{\delta\bar\eta^\Lambda}\; W^\Lambda \frac{\delta}{\delta\eta^\Lambda}\ln W^\Lambda\right] \tag{3}$$

① ②

$\ln W^\Lambda = W^{c,\Lambda}$

$$= \mathrm{Tr}\Big[\overset{①}{\dot C^\Lambda} \underbrace{\frac{\delta}{\delta\bar\eta^\Lambda}W^{c,\Lambda}}_{-\phi}\; \overset{(I.11.1)}{} \underbrace{\frac{\delta}{\delta\eta^\Lambda}W^{c,\Lambda}}_{-\zeta\bar\phi} + \overset{②}{\dot C^\Lambda}\; \underbrace{\frac{\delta^2 W^{c,\Lambda}}{\delta\bar\eta^\Lambda\,\delta\eta^\Lambda}}_{(I.14.1)\; = \;\mathcal{V}^{\Lambda(1,1)}_{\bar\phi\phi}}\Big] \tag{4}$$

Combining (6.4), (6.5), (7.4):

$$\boxed{\dot W^{c,\Lambda} = -(\dot{\bar\eta}^\Lambda,\phi) - (\bar\phi,\dot\eta^\Lambda) + (\bar\phi,\dot C^\Lambda\phi) + \mathrm{Tr}\left[\dot C^\Lambda\left(\mathcal{V}^{\Lambda(1,1)}_{\bar\phi\phi} - \mathcal{G}^{0,\Lambda}\right)\right]} \tag{5}$$

With (7.5) in hand, the flow eq. for Γ follows readily:

$$\Gamma[\phi,\bar\phi] \overset{(I.11.3)}{\equiv} -\mathcal{W}^c[\eta,\bar\eta] - (\bar\phi,\eta) - (\bar\eta,\phi) + (\bar\phi, \mathcal{G}_0^{-1}\phi) \tag{1}$$

$$\dot\Gamma^\Lambda = -\dot{\mathcal W}^{c\Lambda} - (\bar\phi,\dot\eta^\Lambda) - (\dot{\bar\eta}^\Lambda,\phi) + (\bar\phi,\dot C^\Lambda\phi) \tag{2}$$

$$\boxed{\dot\Gamma^\Lambda = -\mathrm{Tr}\left[\dot C^\Lambda\left(\mathcal V^{\Lambda(1,1)}_{\bar\phi\phi} - \mathcal G^{0,\Lambda}\right)\right]} \tag{3}$$

Drop superscript Λ henceforth (for convenience).

Our next task is to express $\mathcal V^{\Lambda(1,1)}_{\bar\phi\phi}$ in terms of Γ and $\mathcal{G}$. Recall:

$$\mathcal V_{\bar\phi,\phi}(\Gamma,\mathcal G^0) \overset{(I.19.1)}{\equiv} \begin{bmatrix} \frac{\delta^2 \mathcal W^c}{\delta\bar\eta\,\delta\eta} & \zeta\frac{\delta^2\mathcal W^c}{\delta\bar\eta\,\delta\bar\eta} \\ \zeta\frac{\delta^2\mathcal W^c}{\delta\eta\,\delta\eta} & \frac{\delta^2 \mathcal W^c}{\delta\eta\,\delta\bar\eta}\end{bmatrix} = \begin{bmatrix} \frac{\delta^2\Gamma}{\delta\bar\phi\,\delta\phi} - \zeta\mathcal G_0^{-1} & \frac{\delta^2\Gamma}{\delta\bar\phi\,\delta\bar\phi} \\ \frac{\delta^2\Gamma}{\delta\phi\,\delta\phi} & \frac{\delta^2\Gamma}{\delta\phi\,\delta\bar\phi} - [\mathcal G_0^{-1}]^t\end{bmatrix}^{-1} \tag{4}$$

FRG-II.9

Write $$\underbrace{\frac{\delta^2\Gamma}{\delta\bar\phi\,\delta\phi} - \gamma_1}_{\equiv \mathcal{U}} \quad \underbrace{+\gamma_1 - S\mathcal{G}_0^{-1}}_{\mathcal{G}_1^{-1}\ (\mathrm{I}.19.5)} = \mathcal{U} + \mathcal{G}^{-1} \tag{1}$$

Similarly: $$\frac{\delta^2\Gamma}{\delta\phi\,\delta\bar\phi} - \gamma_1 + \gamma_1 - (\mathcal{G}_0^{-1})^t = S(\mathcal{U} + \mathcal{G}^{-1})^t \leftarrow \text{transpose} \tag{2}$$

Hence: $$\mathcal{V}_{\bar\phi,\phi} \overset{(8.4)}{=} \begin{bmatrix} \mathcal{U} + \mathcal{G}^{-1} & \frac{\delta^2\Gamma}{\delta\bar\phi\,\delta\bar\phi} \\ \frac{\delta^2\Gamma}{\delta\phi\,\delta\phi} & S(\mathcal{U}+\mathcal{G}^{-1})^t \end{bmatrix}^{-1} = \left[\begin{pmatrix} \mathcal{G}^{-1} & 0 \\ 0 & S(\mathcal{G}^{-1})^t \end{pmatrix} + \begin{pmatrix} \mathcal{U} & \frac{\delta^2\Gamma}{\delta\bar\phi\,\delta\bar\phi} \\ \frac{\delta^2\Gamma}{\delta\phi\,\delta\phi} & S\mathcal{U}^T \end{pmatrix}\right]^{-1} \tag{3}$$

$$= \left[\begin{pmatrix} \mathcal{G}^{-1} & 0 \\ 0 & S(\mathcal{G}^{-1})^t \end{pmatrix} \underbrace{\left[\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} + \overbrace{\begin{pmatrix} \mathcal{G}\mathcal{U} & \mathcal{G}\frac{\delta^2\Gamma}{\delta\bar\phi\,\delta\bar\phi} \\ S\mathcal{G}^T\frac{\delta^2}{\delta\phi\,\delta\phi} & S\mathcal{G}^T\mathcal{U}^T \end{pmatrix}}^{\equiv A}\right]}_{\equiv \tilde{\mathcal{V}} = 1 + A}\right]^{-1} \tag{4}$$

$$\mathcal{V}_{\bar\phi,\phi} = \tilde{\mathcal{V}}^{-1}\begin{bmatrix} \mathcal{G} & 0 \\ 0 & S\mathcal{G}^T \end{bmatrix} \tag{5}$$

FRG-II.10

Now expand:

$$\tilde{\mathcal{V}}^{-1} = (1+A)^{-1} = 1 - A + AA - AAA + \dots \tag{1}$$

$$= 1 - \begin{pmatrix} \mathcal{G}\mathcal{U} & \mathcal{G}\bar\delta^2 \\ S\mathcal{G}^T\delta^2 & S\mathcal{G}^T\mathcal{U}^T \end{pmatrix} + \begin{pmatrix} \mathcal{G}\mathcal{U} & \mathcal{G}\bar\delta^2 \\ S\mathcal{G}^T\delta^2 & S\mathcal{G}^T\mathcal{U}^T \end{pmatrix}\begin{pmatrix} \mathcal{G}\mathcal{U} & \mathcal{G}\bar\delta^2 \\ S\mathcal{G}^T\delta^2 & S\mathcal{G}^T\mathcal{U}^T \end{pmatrix} + \dots \tag{2}$$

$$\tilde{\mathcal{V}}^{-1}_{(1,1)} = 1 - \mathcal{G}\mathcal{U} + (\mathcal{G}\mathcal{U}\mathcal{G}\mathcal{U} + S\mathcal{G}\bar\delta^2\mathcal{G}^T\delta^2) + (\mathcal{G}\mathcal{U}\mathcal{G}\mathcal{U}\mathcal{G}\mathcal{U} + \dots) \tag{3}$$

brackets group order of expansion

To evaluate $\mathcal{U}$, δ^2, $\bar\delta^2$, we need explicit expression for Γ:

$$\gamma_m(k_1',\dots,k_m';k_1,\dots,k_m) \overset{(\mathrm{I}.13)}{=} \frac{\delta^m}{\delta\bar\phi_{k_1'}\cdots\delta\bar\phi_{k_m'}}\frac{\delta^m}{\delta\phi_{k_m}\cdots\delta\phi_{k_1}}\Gamma[\phi,\bar\phi]\Big|_{\phi=\bar\phi=0} \tag{4}$$

$\Rightarrow$ [pulling $\frac{\delta}{\delta\phi_k}$ past $\bar\phi_{k'}$ gives S]

$$\Gamma = \sum_{m=0}^{\infty}\frac{S^m}{(m!)^2}\sum_{k_1',\dots,k_m'}\sum_{k_1,\dots,k_m}\gamma_m(k_1',\dots,k_m';k_1,\dots,k_m)\,\bar\phi_{k_1'}\cdots\bar\phi_{k_m'}\,\phi_{k_m}\cdots\phi_{k_1} \tag{5}$$

since γ's are antisymmetric w.r.t. $k_i' \leftrightarrow k_j'$ and/or $k_i \leftrightarrow k_j$

$$\boxed{\mathcal{U}_{q'q} \overset{(9.1)}{=} \underbrace{\frac{\delta^2\Gamma}{\delta\bar\phi_{q'}\,\delta\phi_q}}_{\text{shifts } m \text{ by } 1} - \gamma_1 \quad \leftarrow \text{subtracts off } m=1 \text{ contribution}}$$

$$\boxed{= \sum_{m=1}^{\infty}\frac{\delta^m}{(m!)^2}\sum_{k'_1,\dots,k'_m}\;\sum_{k_1,\dots,k_m}\gamma_{m+1}(k'_1,\dots,k'_m,q';k_1,\dots,k_m,q)\,\bar\phi_{k'_1}\cdots\bar\phi_{k'_m}\,\phi_{k_m}\cdots\phi_{k_1}} \quad (1)$$

Note: $\mathcal{U}$ is at least of order $\bar\phi\phi$ and does not depend on γ_0, γ_1.

Now we are finally in a position to write down flow equations for γ_m, by expanding (8.3), and using (10.3) on RHS:

$$\dot\Gamma^\Lambda \overset{(8.3)}{=} -\mathrm{Tr}\Big[\dot C^\Lambda\big(\underbrace{V^{\Lambda(1,1)}_{\bar\phi\phi}}_{(10.3)\;[1-\mathcal{G}\mathcal{U}+\dots]\mathcal{G}} - \mathcal{G}^{0,\Lambda}\big)\Big] \quad (2)$$

$m=0$:

"zero-particle vertex function":

$$\boxed{\dot\gamma_0 = -\mathrm{Tr}\big[\dot C^\Lambda(\mathcal{G}-\mathcal{G}^{0,\Lambda})\big]} \quad (3)$$

$m=1$: "single-particle vertex function"

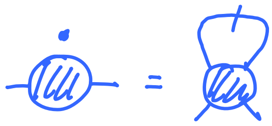
Collect terms linear in $\bar\phi_{k'}\phi_k$ from (11.2): (or take $\left.\frac{\delta^2\,(11.2)}{\delta\phi_{k'}\,\delta\phi_k}\right|_{\phi=0=\bar\phi}$)

$$\dot\gamma_1(k',k) = \sum_{q',q}(\mathcal{G}\dot C^\Lambda\mathcal{G})_{qq'}\,\underbrace{\gamma_2(k',q';k,q)}_{\text{shorthand } \equiv [\gamma_2(k',\cdot\,;k,\cdot)]_{q'q}} \quad (1)$$

Define: $S \equiv \mathcal{G}\dot C^\Lambda\mathcal{G} = \mathcal{G}[\mathcal{G}^{0,\Lambda}]^{-1}\mathcal{G}$ ≡ (2)

diagrammatic convention

$$\boxed{\dot\gamma_1(k',k) = \mathrm{Tr}\big[S\,\gamma_2(k',\cdot\,;k,\cdot)\big]} \quad (3)$$



Flow eq. for γ_1 expressed in terms of γ_2 (which contains only two-particle-irreducible diagrams). NICE!